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Transient Climate Response to Cumulative Emissions (TCRE) As A Reduced-form Climate Model

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ABSTRACT

This study examines the paradox that the path of realized global warming by a given year is linear in cumulative emissions of carbon dioxide even though the corresponding eventual equilibrium warming is only logarithmic in atmospheric concentration. It then provides parameters for this linear relationship using the high-emissions scenario of the Intergovernmental Panel on Climate Change. These estimates constitute a transparent reduced-form climate model that can be useful for analyzing the economic costs and benefits of alternative abatement scenarios.

Keywords: Transient Climate Response, Integrated Assessment Models
(JEL Q54)

Introduction

Integrated assessment models (IAMs) examine costs and benefits of carbon dioxide abatement, with the objective of identifying optimal abatement policy or alternatively assessing net costs of meeting specified climate targets. These models apply energy-economic modules to calculate costs of alternative policies. They apply compact climate modules to determine future global warming under alternative policies. After specifying likely damages associated with alternative levels of warming, the models provide a comparison of abatement costs to the "benefits" of damage avoided by abatement policies.

The three leading IAMs are the DICE model of William Nordhaus (2020), the FUND model of Richard Tol (Waldhoff, Anthoff, Rose, and Tol, 2014), and the PAGE model of Nicholas

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Hope (2011).[2] The 2010 US Interagency study on the social cost of carbon based its estimates on these three IAMS (Interagency Working Group, 2010).

For any given concentration of atmospheric carbon dioxide, there is a decades-scale lag from "transient" global warming actually realized by the year this concentration is reached to the eventual equilibrium warming expected at this concentration. An "ocean thermal lag" occurs as, in an initial phase, part of the warming potential is absorbed through the warming of the deep ocean. The DICE model explicitly models two "boxes": one for the deep ocean and the other for the atmosphere, land surface, and ocean surface. Its thermal lag occurs through a term that captures heat transfer from the upper to the lower box. This transfer diminishes as the temperature differential between the two boxes declines from warming of the low ocean. The other two models have a single-box structure. Calel and Stainforth (2017) find that "even with identical economies, these three IAMs produce substantially different climate change forecasts (p. 1212).[3]

This study suggests that there is a simple, transparent alternative to the climate modules in these three IAMs: the linear relationship between observed (transient) warming and cumulative anthropogenic emissions (TCRE) identified in the most recent IPCC report (IPPC, 2013). The discussion first seeks to explain the apparent contradiction between linear warming from cumulative emissions and logarithmic radiative forcing by atmospheric concentration of CO_2. It then examines projections of warming under the high emissions scenario (RCP8.5) of the IPCC's fifth assessment report using the TCRE linear model.

Radiative Forcing and Global Warming

Radiative forcing occurs when there is a perturbation that causes the outbound radiation from the earth to differ from the amount of inbound radiation that reaches the earth. The earth's natural greenhouse effect (from water vapor, carbon dioxide and other greenhouse gases) prevents the full amount of radiation received from the sun from being returned to space. However, the warmer a planet is, the more outbound (longwave) radiation it emits. As examined in Appendix A, the earth is warmed enough by the natural greenhouse effect to arrive at a temperature high enough to generate the same amount of outgoing radiation as incoming radiation. Without the natural greenhouse effect, earth's temperature would be -18^{o}C instead of 15^{o}C.[4]

An increase in the greenhouse effect from its natural level as a consequence of man-made emissions of carbon dioxide and other greenhouse gases has the effect of trapping more outbound radiation, and hence requiring a still higher temperature of earth to force outbound

[2] The acronyms, respectively, stand for: Dynamic Integrated model of the Climate and the Economy; Climate Framework for Uncertainty, Negotiation and Distribution; and Policy Analysis of the Greenhouse Effect.
[3] Thus, the authors find that by 2300 there is a range among the three models of 1.7^{o}C for warming under the high (RCP8.5) emissions scenario of the IPCC (2013).
[4] Note however that this standard estimate depends on unchanged albedo (fraction of the sun's radiation reflected to space instead of entering earth's atmosphere) in the absence of the atmosphere; see Appendix A.

radiation to return to equal the level of inbound radiation. Even though the change in radiation is small relative to the total two-way flow, it is sufficient to pose potentially severe future warming, by as much as 12.6°C above 1986-2005 levels at the 95th percentile by 2300 in the IPCC's high-emissions scenario (IPCC, 2013, p. 1055).[5]

Logarithmic Radiative Forcing of CO_2

In the 1820s French mathematician Joseph Fourier calculated that the earth was much warmer than could be explained by its proximity to the sun, and attributed this phenomenon to the trapping of outbound infrared radiation by gases in the atmosphere.[6] In the 1860s Irish physicist John Tyndall conducted laboratory experiments identifying water vapor as the most important one, followed by carbon dioxide and methane. In 1896 Swedish scientist Svante Arrhenius used observations by US astronomer Samuel Langley on radiation received by the earth from the full moon, at differing angles and hence concentrations of water vapor and CO_2, to measure the influence of carbon dioxide on ground temperature. He found: that "if the quantity of carbonic acid increases in geometric progression, the augmentation of the temperature will increase nearly in arithmetic progression (Arrhenius, 1896, 267).

Water vapor and carbon dioxide are transparent to incoming solar radiation in the visible spectrum of 0.4 to 0.7 microns wavelength, but opaque to outbound infrared radiation in most bands (especially 12-20 microns for CO_2, and 5-7 as well as 15-30 for water vapor) (CSI, 2020). The blocking effect of additional carbon dioxide is less than linear as the opaque bands become saturated.

As discussed in Appendix A, in the first IPCC report (IPCC, 1990) and its successor, the IPCC placed the direct radiative forcing of a doubling of carbon dioxide at 4.37 Wm^{-2} (Watts per square meter). The Third Assessment Report in 2001 reduced the estimate to 3.7 Wm^{-2}(IPCC, 2001, 357). The basic logarithmic structure originally suggested by Arrhenius remains unchanged, however.

AR5 and Linear TCRE Warming

In its Fifth Assessment Report (AR5), the IPCC (2013) introduced the measure "transient climate response to cumulative carbon emissions," with the acronym "TCRE." It stated:

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[5] Incoming radiation from the sun is approximately 342 Watts per square meter of earth's surface (Salawitch et al, 2017, 8). Of this amount, about 30 percent is reflected back to space by the "albedo" provided primarily by clouds but also by aerosols, air, water vapor, and the earth's surface. Radiation reaching the earth's surface is approximately 240 Wm^{-2}. In comparison, the increase in radiative forcing from the man-made greenhouse effect has currently reached about 2 Wm^{-2}, but is projected at 8 Wm^{-2} by 2100 and 12 Wm^{-2} by 2300 in the highest (business-as-usual, RCP8.5) scenario considered by the IPCC (2013, p. 1046).

[6] AIP (2020) describes contributions and controversies in the development of science on the warming effect of carbon dioxide.

This metric is useful to determine the allowed cumulative carbon emissions for stabilization at a specific global temperature. TCRE is defined as the annual mean global surface temperature change per unit of cumulated CO_2 emissions, usually 1000 PgC [GtC], in a scenario with continuing emissions … It considers physical and carbon cycle feedbacks … [but not] release of methane hydrates or large amounts of carbon from permafrost. The assessment based on climate models as well as the observed warming suggests that the TCRE is *likely* 0.8^oC to 2.5^oC per 1000 PgC … for cumulative CO_2 emissions less than about 2000 PgC until the time at which temperatures peak. … [TCRE] ignores non-CO_2 forcings … (IPCC, 2013, 1112).[7]

The principal driver of long-term warming is total emissions of CO_2 and the two quantities are approximately linearly related. … To limit the warming caused by anthropogenic CO_2 emissions alone to be *likely* less than 2^oC relative to the period 1861-1880, total CO_2 emissions from all anthropogenic sources would need to be limited to a cumulative budget of about 1000 PgC since that period. About half "[445 to 585 PgC]" of this budget was already emitted by 2011. Accounting for projected warming effect of non-CO_2 forcing, a possible release of GHGs [greenhouse gases] from permafrost or methane hydrates, or requiring a higher likelihood of temperatures remaining below 2^oC, all imply a lower budget. (IPCC, 2013, 1033).

A large fraction of climate change is largely irreversible on human time scales, unless net anthropogenic CO_2 emissions were strongly negative over a sustained period. For scenarios driven by CO_2 alone, *global average temperature is projected to remain approximately constant for many centuries following a complete cessation of emissions* [emphasis added]. (IPCC, 2013, 1033)

The Fifth Assessment Report maintained the range of 1.5^oC to 4.5^oC for Equilibrium Climate Sensitivity (ECS), the eventual equilibrium warming from a doubling of carbon dioxide above pre-industrial levels after allowing for ocean thermal lag. For the corresponding concept of Transient Climate Response (TCR), the warming that occurs already by the time CO_2 doubles (under the assumption of steady increase at 1 percent per year), AR5 placed the range at 1^oC to 2.5^oC. (IPCC, 2013, 1033). It noted that "for low to medium estimates of climate sensitivity, the TCRE is nearly identical to the peak climate response" (p. 1108).

AR5 does not discuss why TCRE should show linear warming as a function of cumulative emissions when the relationship of radiative forcing to atmospheric concentration is only logarithmic. Instead it implicitly treats TCRE linearity as an artifact that emerges from numerous climate model simulations.

[7] 1 petagram (Pg) ≡ 10^{15} g = 10^{12} kg = 10^9 t ≡ 1 gigaton (Gt)

Explaining the Paradox of Linearity

One consideration in explaining the paradox of linearity in TCRE warming is that the cumulative function of a series inherently rises exponentially relative to the underlying series. Suppose *y* is the cumulative function of *x*. Suppose $x = a + bt$, a linear function of time. Then y_t will be the integral of x_t , or: $at + [b/2]t^2$. The cumulative function rises over time in a manner that involves one power more than the underlying series (t^2 rather than just t). Cumulative emissions are inherently exponential relative to the path of emissions.

However, it is atmospheric stock of carbon dioxide, or concentration in parts per million volume, that matters for radiative forcing. A committee commissioned by the National Academy of Sciences (NAS, 2016) observed that:

> The weakening of the land and ocean carbon sinks as a result of warming increases the atmospheric residence time of CO_2 … giving rise to a convex relationship between cumulative carbon emissions and atmospheric CO_2 concentrations. When the convex relationship between emissions and concentrations is combined with the concave relationship between concentrations and forcing, the result is a coincidental cancellation that results in a nearly linear relationship between cumulative CO_2 emissions and radiative forcing (chapter 3, p. 16).

Figure 1 reports the time path of carbon dioxide emissions projected by the IPCC in the business-as-usual scenario "RCP8.5" in AR5 (see Appendix B), along with the corresponding cumulative emissions and atmospheric stock (all expressed in gigatons of carbon). Shown against the right-hand scale, carbon dioxide emissions rise from 10 GtC in 2010 to 24 GtC by 2060 and then more gradually to 29 GtC by 2100.[8] Cumulative emissions (left scale) are set at 515 GtC in 2011 (the mid-point of the IPCC range noted earlier) and thereafter simply add each subsequent year's emissions. Cumulative emissions rise exponentially, as expected, reaching about 2400 GtC by 2100 – far exceeding the total carbon "budget" of 1000 GtC for limiting warming to 2^oC.

The figure also shows the atmospheric stock of carbon corresponding to this scenario, derived from the IPCC's time path for the concentration of carbon dioxide (IPCC, 2013, 1103).[9] Atmospheric stock rises from 782 GtC in 2000 to 887 GtC in 2011, 1260 GtC by 2060, and 2030 GtC by 2100. There is an evident acceleration in the rate of increase in the atmospheric stock after 2060.

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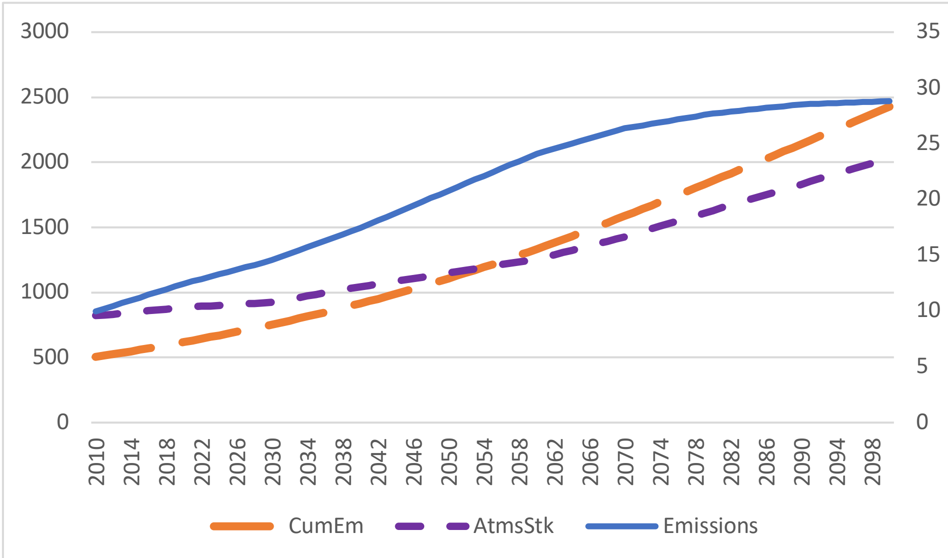
[8] Annual anthropogenic emissions of CO_2 are from Meinshausen et al (2011), with data available at: https://www.iiasa.ac.at/web-apps/tnt/RcpDb. In 2010 the total of 10 GtC includes 1 GtC from land use change. This component falls to 0.6 GtC by 2050 and zero by 2100. Note that 1 GtC = 3.67 $GtCO_2$ (from [2x16+12]/12 based on atomic weights).

[9] Each part per million volume of carbon dioxide in the atmosphere corresponds to a stock of 2.12 GtC. Thus, in 1994 the atmospheric stock was estimated at 762 GtC, and concentration was 359 ppmv (parts per million volume), a ratio of 2.12 GtC/ppmv. By 2011 atmospheric stock had reached an estimated 829 GtC and concentration 390 ppmv, for a ratio of 2.126. See IPCC (2007, 575; 2013, 471), and NOAA (2020).

Figure 1
Emissions (rhs), Cumulative Emissions, and Atmospheric Stock of Carbon (RCP8.5, in GtC, 2010-2100)



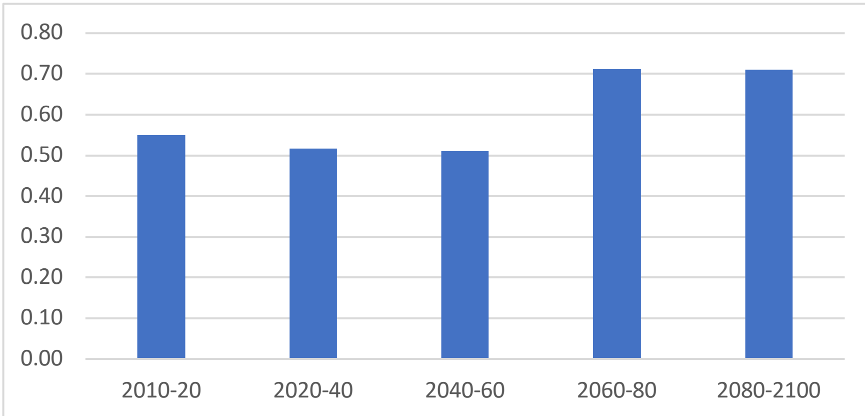
Source: IPCC (2013), Meinshausen et al (2011), and author's calculations

A simple calculation of the average atmospheric retention ratio (ARR) based on the change in atmospheric stock divided by the change in cumulative emissions does show the rising retention rate after 2060, supporting the NAS (2016) view. Thus, whereas only about 50 percent of total emissions in from 2010 to 2060 show up in rising atmospheric stock, the ARR rises to about 70 percent in 2060-2100 (figure 2). The overall effect for atmospheric stock is a sufficiently rapid rise that the logarithm of the stock rises approximately linearly (figure 3). As a consequence, radiative forcing rises linearly.[10]

Appendix C examines the linearity paradox further by taking as its point departure a framework solely considering constant emissions (at their 2010 level) and their cumulation over a 10-year period. The path of the logarithm of the resulting "stock" series shows a rapid drop-off in the annual increase in the logarithm of stock, seemingly consistent with the intuition of benignancy thanks to merely logarithmic forcing. However, taking account of the 'legacy" of an atmospheric stock that is many times larger than the annual emissions sharply reduces the pace of decline in the path of the logarithm of the (legacy-included) stock. Further taking account of rising atmospheric retention and rising emissions in the numerical illustration reverses the declining marginal impact to a path of rising marginal impact.

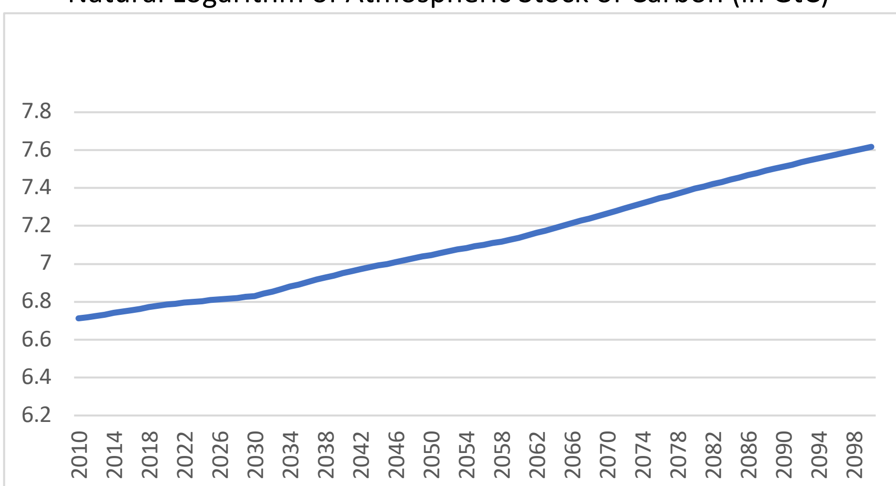
[10] The radiative forcing equation uses the logarithm of the ratio of concentration (and hence atmospheric stock) to its pre-industrial level. However, the logarithm of a ratio is the logarithm of the numerator less the logarithm of the denominator, so the logarithm of the ratio to pre-industrial stock will be a line with the same slope as the logarithm of the stock and an intercept equal to the difference between the logarithm of the stock at the beginning of the period considered and the logarithm of the pre-industrial stock. (The pre-industrial stock was 280 ppmv x 2.12 GtC/ppmv = 593.6 GtC., yielding a natural logarithm of 6.39.)

Figure 2
Atmospheric Retention Ratio for CO_2 Emissions



Source: IPCC (2013) and author's calculations

Figure 3
Natural Logarithm of Atmospheric Stock of Carbon (ln GtC)[a]



a. RCP 8.5 scenario
Source: IPCC 2013 and author's calculations.

A reduced-form climate model

Overall, linear warming against cumulative emissions in the TCRE approach appears to be verified despite the seeming paradox from merely logarithmic forcing of CO_2. To arrive at a simple, reduced-form climate model using TCRE then requires deciding on the most appropriate value for the constant multiplicative parameter within the IPCC's range of 0.8°C to 2.5°C warming per 1000 GtC. Call this parameter τ. Then $W = \tau Z$ where W is transient warming above pre-industrial levels and Z is cumulative carbon dioxide emissions in GtC.

The Fifth Assessment Report reported that the Coupled Model Intercomparison Project 5 (CMIP5) survey of 39 climate models found that in scenario RCP8.5, global average temperature change above average temperature in 1986-2005 would amount to 0.46^{o}C by 2039 and 4.1^{o}C by 2100. Because the TCRE relationship is specified against the same pre-industrial base as the Equilibrium Climate Sensitivity benchmark, it is necessary to adjust the CMIP5 estimates to warming against a pre-industrial base.

The IPCC (2018, 51) places warming above pre-industrial levels at 1.0^{o} C in 2017. AR5 reports the CMIP5 estimate for 2017 warming above the 1986-2005 average as 0.62^{o}C (IPCC (2013, 21, Figure SPM.7). The CMIP5 estimates can thus be converted to warming above pre-industrial levels by adding 0.38^{o}C to each year's estimated warming above the 1986-2005 average temperature.

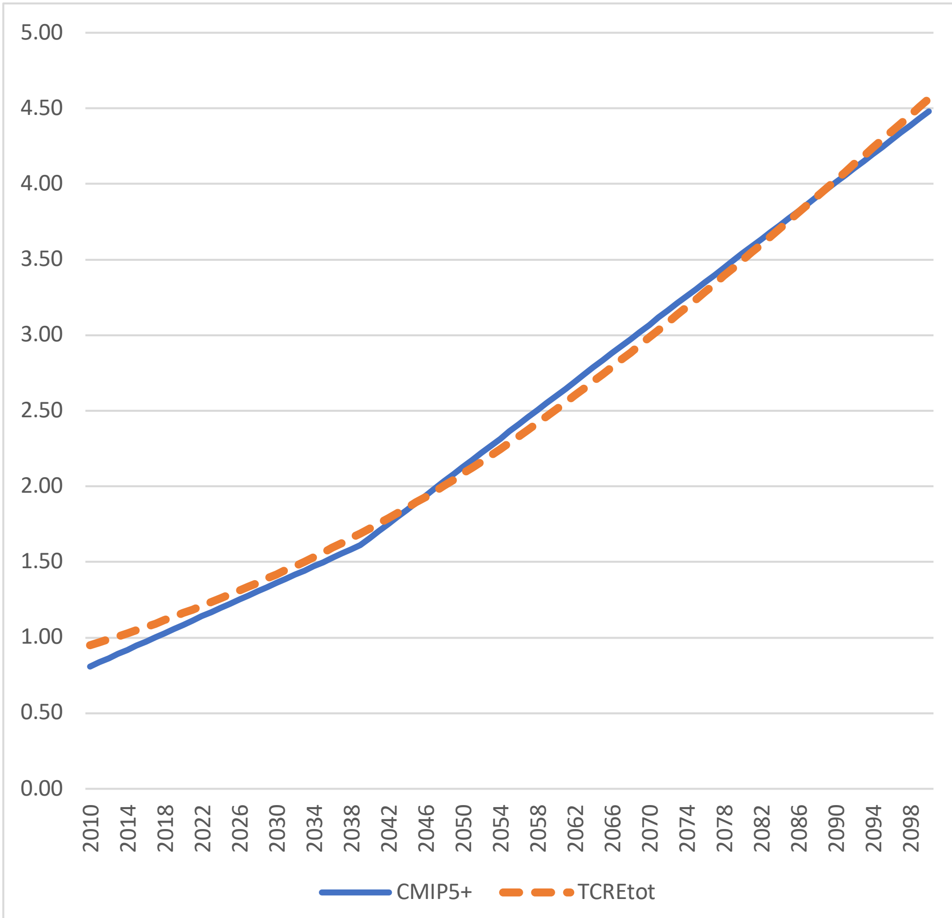
A key question is how to treat emissions of, and warming from, other greenhouse gases (primarily methane, nitrous oxide, and hydroflouorocarbons) within a TCRE framework, considering that it is formulated referring solely to carbon dioxide. In the RCP8.5 scenario, emissions of other greenhouse gases turn out to be broadly a constant 20 percent of total CO_2-equivalent emissions through 2010-2100 (see Appendix B). On this basis, it is useful to formulate a "broad" TCRE relationship in which the parameter relating warming to cumulative emissions of carbon *includes* the effect of other greenhouse gases.

Figure 4 shows that a relatively close fit is obtained by applying this type of "broad" TCRE, with the parameter τ set at 1.88^{o}C per 1000 GtC cumulative carbon dioxide emissions.[11] This value is higher than the 1.65^{o} midpoint of the IPCC's range of 0.8^{o}C to 2.5^{o}C per thousand GtC cumulative emissions, as would be expected considering that the "broad" parameter includes the effect of other greenhouse gases.

[11] CMIP5+ in figure 4 adds 0.38^{o}C to the CMIP5 path of warming to convert to warming above pre-industrial levels. Broad TCRE (TCREtot) in the figure refers to inclusion of other greenhouse gases in the parameter for impact (but without their explicit incorporation of the CO_2 equivalent into the measure of cumulative carbon dioxide emissions). The estimate of τ = 1.88 for the (broad) TCRE parameter minimizes the sum of squared residuals of TCREtot from CMIP5+.

Figure 4

Warming Above Pre-industrial Levels in CMIP5 and Broad TCRE with τ = 1.88°C/1000 GtC Cumulative Carbon Emissions (RCP8.5) (Degrees Celsius)



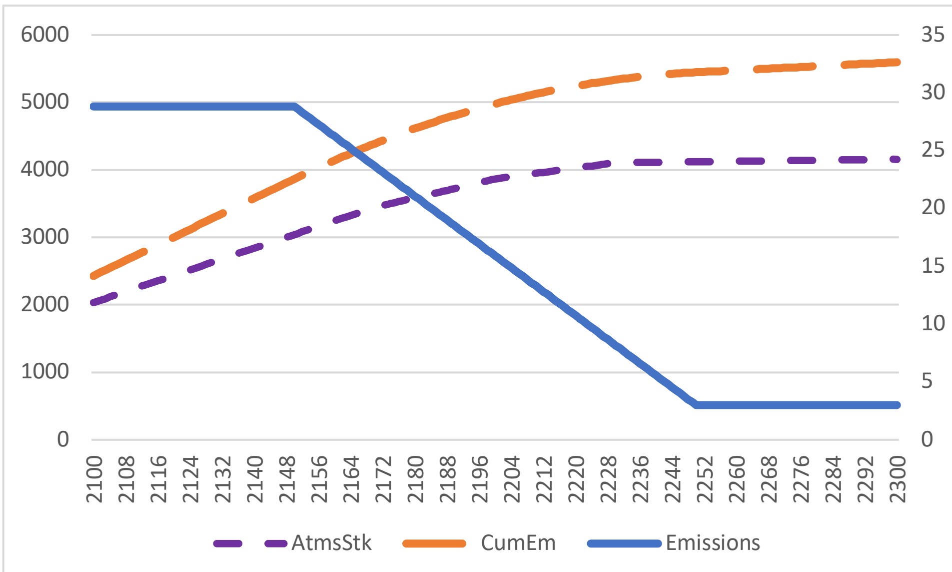
Source: IPCC (2013, 21) and author's calculations

Emissions and TCRE for 2100-2300

In its Fifth Assessment Report the IPCC (2013, chapter 12) for the first time extended its projections through 2300. This longer-term horizon is crucial to a meaningful analysis of the economics of global warming (Cline, 1992). As discussed in Appendix B, the AR5 assumption for the RCP8.5 scenario was that carbon dioxide emissions would remain constant at about 30 GtC annually in the period 2100-2150, and would then decline linearly to a low plateau of 3 GtC by 2250.

Figure 5 shows the results of carrying out the same sequence of calculations for 2100-2300 as those used for the paths reported for 2010-2100 in figure 1. Emissions (right-hand scale) follow the AR5 RCP8.5 path as provided in detail in Meinshausen et al (2011, 228). Cumulative emissions begin at 2427 GtC in 2100 and add each year's emissions, bringing the total to 5596 by 2300. Atmospheric stock is again derived from the path reported for concentration (IPCC, 2013, 1103), and rises from 2030 GtC in 2100 to 4155 GtC in 2300.

Figure 5
Emissions (rhs), Cumulative Emissions, and Atmospheric Stock of Carbon (RCP8.5, in GtC, 2100-2300)



Source: IPCC (2013), Meinshausen et al (2011), and author's calculations

For its part, the atmospheric retention ratio implied by comparison of the change in atmospheric stock against emissions remains approximately constant at approximately 0.7 in the period 2100-2230 but then abruptly declines to about 0.2 for the rest of the 23rd century.

Figure 6 shows the CMIP5 central path for warming above pre-industrial levels from 2100 through 2300.[12] The figure also shows the predicted TCRE warming, again a version incorporating all greenhouse gases (TCREtot). This time, however, the parameter τ used to calculate TCREtot (which equals τ x cumulative emissions in GtC) is allowed to vary rather than being forced to be a single constant over the 200 year period.

A regression of the ratio of warming to cumulative emissions yields the following relationship:

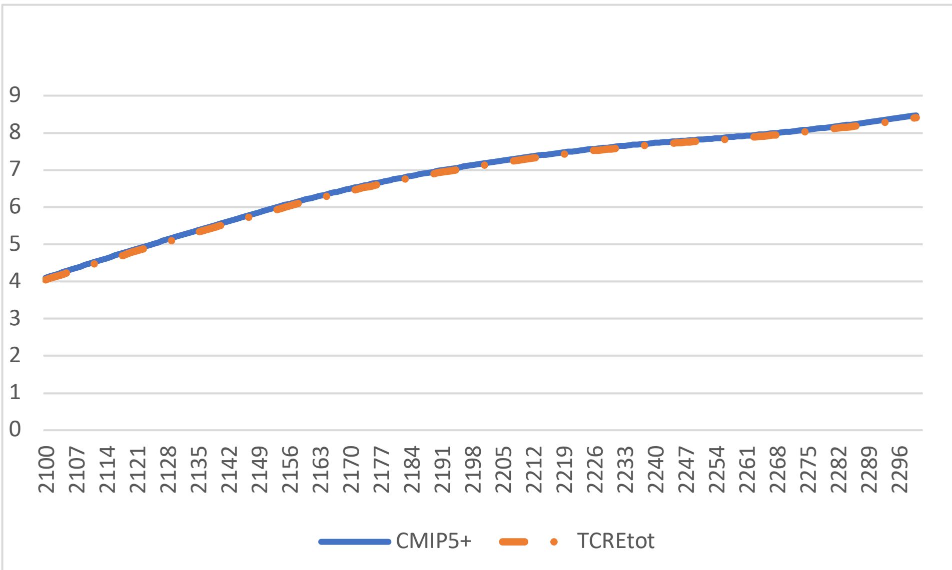
$$1)\ \tau = 1.67 - 0.00396\ t + 0.0000155\ t^2$$

[12] Once again the AR5 path shown in IPCC (2013, 1054) is adjusted upward by 0.38°C to convert it from the average 1986-2005 temperature base to a pre-industrial base (CMIP+ in the figure). Note also that there is a hiatus at 2100 in which the path from 2010 to 2100 ends 0.36°C higher than the figure for 2100 in the 2100-2300 path. This discontinuity is "due to different numbers of models performing the extension runs beyond the 21st century and [has] no physical meaning." Note that 39 models are used for the first period and 12 for the second.

where t is the year, from 1 for 2100 to 200 for 2300.[13] This broad TCRE parameter has a value of 1.67 in 2100, declines steadily to a trough of 1.42 by 2225 and then gradually returns to 1.50 by 2300. The initial difference from the 2010-2100 value of τ = 1.88 is explained by the discontinuity at 2100 in the models included in CMIP5.[14] The decline from 2100 to 2300 reflects in part the rising share of CO_2 in the combined radiative forcing of the most important greenhouse gases over this period (see Appendix D).

Figure 6

Warming Above Pre-industrial Levels in CMIP5 and Broad TCRE, 2100-2300 (RCP8.5; °C)



Source: IPCC (2013, 1054) and author's calculations

Extreme Values

The warming projections shown in figures 4 and 6 are from the central values in the CMIP5 exercise. The IPCC (2013, 1055) also reports the corresponding 5th percentile low and 95th percentile high values. These are respectively 0.7 and 1.3 times the central values for 2081-2100, with corresponding multiples at 0.51-1.51 for 2181-2200 and 0.38-1.62 for 2281-2300.

Applying these relationships to the above estimates for the broad TCRE parameter τ, with a central value of τ at 1.88 in the 21st century and the value from equation 1) in 2100-2300, yields the following low-high extremes. Predicted warming above pre-industrial levels would be 2.8-5.3°C above pre-industrial levels by 2091; 3.5-10.5°C by 2191; and 3.1-13.4°C by 2291.

[13] T-statistics are 549 for the intercept, -57 for t, and 47 for t^2. Adjusted R^2 is 0.96.
[14] See note 12.

TCRE in a Lower Emissions Path

In the IPCC's emission path RCP6.0, carbon dioxide emissions remain flat at 10.4 GtC from 2010 through 2040. They then rise to a peak of 17.5 GtC by 2080, but decline thereafter to 13.9 GtC by 2100 (Meinshausen et al, 2011, 228). Cumulative emissions reach 1678 GtC by 2100, only about two-thirds the level reached in RCP8.5. Atmospheric concentration reaches 661 ppm by 2100, and warming above pre-industrial levels reaches 1.57°C by 2150 and 2.83°C by 2100 (IPCC 2013; 21 (Figure SPM7), 1011 (Figure 11.25)). The IPCC projections do not extend beyond 2100 for RCP6.0.

During the period 2010-2100, in RCP6.0 the broad TCRE coefficient τ stands at 1.60 in 2010, rises to a peak of 1.74 by 2060, and then eases to 1.69 by 2100.[15] The average over the full period is 1.70. By implication, the use of this more moderate emissions scenario reduces the broad τ coefficient by about 10 percent, a change not sufficient to change the overall qualitative implications of the analysis based on RCP8.5.

Comparison to DICE, FUND, and PAGE

Figure 6 shows that the broad TCRE model estimated here places RCP8.5 warming above pre-industrial levels at a central estimate of 8.4°C in 2300.[16] Calel and Stainforth (2017, 1206) report that the corresponding estimates are 8.4°C in PAGE09, 9.2°C in DICE-2013R, and 9.8°C in FUND3.9. Accordingly, the TCRE-based reduced-form model suggested in this study yields estimates in the same broad range as those of the three leading IAMs used in climate-economic analysis. Although it is at the conservative flank compared to these three models, it replicates the central estimates of the CMIP5 atmospheric science models.

Overview

This study first demonstrates the plausibility of a linear relationship between cumulative carbon dioxide emissions and transient warming despite the underlying logarithmic relationship between warming and atmospheric concentration of carbon dioxide. As developed in Appendix C, the largest influence resolving this paradox is that when the existing stock of natural and anthropogenic carbon dioxide already in the atmosphere is taken as the point of departure (legacy effect), the increments from even high emissions paths turn out to be small enough in comparison that annual changes are not a large percent of the atmospheric stock.

Correspondingly, the logarithmic relationship of radiative forcing to atmospheric stock yields successive marginal warming increments that do decline, but not a rapid pace. Once a rising atmospheric retention rate and rising rather than constant emissions are taken into

[15] Author's calculations.

[16] From equation 1), in 2300 τ = 1.513. Cumulative emissions stand at 5561 GtC. Predicted warming above pre-industrial levels is 1.513 x 5561 x 0.001 = 8.41°C.

account, marginal warming rises rather than falling. Cumulative anthropogenic emissions rise somewhat more rapidly than total atmospheric stock, such that the overall effect is a linear relationship between cumulative emissions and warming.

The reduced-form TCRE model calibrated in this study on the basis of the CMIP5 survey of climate models yields the following results, using the RCP8.5 business as usual scenario of high emissions. In the 21st century: $W_t = 1.88\ CE_t$ where W is realized (transient) warming above pre-industrial levels, in degrees Celsius, by year t (with 2010 = 1) and CE is cumulative anthropogenic emissions of carbon dioxide expressed in gigatons of carbon. For the 22nd and 23rd centuries, the corresponding estimate is: $W_t = (1.67 - 0.00396\ t + 0.0000155\ t^2)\ CE_t$, this time with $t = 1$ in 2100. Both of these estimates represent a "broad" concept of the warming coefficient, designed to include the influence of other greenhouse gases.

An important feature of this reduced-form model is that it provides a specific central estimate for the warming impact parameter τ rather than only recognizing a broad possible range for it, reported as 0.8°C to 2.5°C per 1000 GtC of cumulative emissions by the IPCC (2013, 1112). Moreover, the decline of τ during 2100-2300 (from 1.67 at the outset of this period to 1.50 at its end), reflects the rising relative role of carbon dioxide compared to other greenhouse gases over these two centuries.

The reduced-form TCRE model developed here is transparent and easy to implement. It may be useful for analyses of the social cost of carbon dioxide, as it provides a simple alternative to the differing climate modules developed in the three principal Integrated Assessment Models that have been used for such analysis. By design, this reduced-form model adheres closely to the CMIP5 outcomes (see figures 4 and 6), thereby representing central values from the leading coupled atmosphere-ocean general circulation models. It avoids challenges of accessibility and need for "thousands" of simulations characterizing FUND and PAGE, and avoids sensitivity to the specific calibrations of the key parameters for heat transfer between the deep-ocean box and surface-atmosphere box in DICE.[17]

[17] See Calel and Stainforth (2017; 1202, 1210).

Appendix A

Radiative Forcing of Carbon Dioxide

By the Stefan-Boltzmann law, the radiative energy emitted by a "black body" is proportional to the fourth power of its temperature in degrees Kelvin (T).[18] The constant term in this proportional relationship is: $\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm}^{-2} / T^4$. Define S as the inbound solar radiation reaching the top of the earth's atmosphere. Define α as the fraction of this radiation reflected back to space by earth's "albedo" (primarily from reflection by clouds but also air, water vapor, aerosols, and earth's surface). Because a sphere's surface is four times as large as its disk image, the incoming radiant energy per square meter reaching the earth's surface is $(1-\alpha)\ S/4$.[19]

If there were no natural greenhouse effect, such that the entirety of the sun's energy reaching the earth's surface were emitted outward again in long-wave radiation, earth's temperature T_0, and outward radiation R^*_{OL} (in Watts per square meter) would be expected to be as follows:

$$A1)\ R^*_{OL} = \sigma T_0^4 = (1 - \boldsymbol{\alpha})(\mathrm{S} \div 4)$$

where the final right side of equation A1) is the incoming solar radiation per square meter of the earth's surface. Earth's equilibrium temperature would thus be:

$$A2) T_0 = \left[\frac{(1-\alpha)S}{4\sigma}\right]^{1/4}$$

Salawitch et al (2017, 8) place S at 1370 Wm^{-2} and α at 0.3. Using these values, in the absence of the natural greenhouse effect earth's temperature would be 255°K (-18°C), or 33 degrees colder than earth's actual temperature of 288°K (15°C). This is a standard estimate for earth's temperature in the absence of any atmosphere, although that interpretation requires the assumption that albedo would remain unchanged even though the atmosphere contributes 87 percent of albedo (Stephens et al, 2015, 148).[20]

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[18] Named for two Austrian physicists, Josef Stefan and Ludwig Boltzmann. In 1879 Stefan identified this relationship experimentally, and in 1884 Boltzmann derived the same law on the basis of thermodynamics. (Encyclopedia Brittanica.)

[19] A sphere with radius "r" has a disk image that is the area of the circle at its equator, or πr^2. The volume of a sphere is (4/3) πr^3, which can be demonstrated by integrating a series of disks parallel to the axis. Slightly increasing the radius and volume of a sphere would increase its surface area, so the surface area can be seen as the derivative of the sphere's volume with respect to the radius. This derivative is 4 πr^2, equivalent to the area of 4 disks. Note that with a radius of 6,371 km, the earth's surface has 510 trillion square meters.

[20] At unchanged albedo, $T_0 = \{[0.7 \times 342.5]/[5.67 \times 10^{-8}]\}^{0.25} = \{[239.75/5.67] \times 10^8\}^{0.25} = 42.28^{0.25} \times 10^2 = 255$. If instead albedo falls to zero, equation A2) yields T_0 = 278.8°K. Arguably increased extent of snow and ice could

The fact that earth's pre-industrial temperature was 33°C higher than explained by the sun's energy net of albedo means that a sufficient portion of potential outbound longwave radiation was being trapped by natural greenhouse gas concentrations to allow a higher temperature for earth without higher inbound solar radiation. Taking the derivative of equation A1) yields $dR/dT = 4\, \sigma T^3$. Evaluated at 255°K, this change in radiative forcing equals 3.76 Wm^{-2} per degree Kelvin. Taking the inverse, the corresponding parameter is that *an increase of 1* Wm^{-1} *in radiative forcing yields a temperature increase of 0.266°C*.

Bony et al (2006, p. 3475) also note that the value of 3.8 $Wm^{-2}K^{-1}$ radiative forcing for an extra degree K is obtained "simply … by equating global mean OLR [Outbound Longwave Radiation] to σT^4 and by assuming an emission temperature of 255K" (p. 3475). However, they indicate that simulations of global climate models instead place this parameter at only 3.2 Wm^{-2} for 1 degree warming, yielding an *alternative estimate that the temperature increase is 0.31°C for 1* Wm^{-2} additional radiative forcing.

As indicated in the main text, radiative forcing has long been viewed as bearing a logarithmic relationship to the atmospheric concentration of carbon dioxide. The First Assessment Report (FAR) by the IPCC in 1990 adopted the following relationship of radiative forcing to the natural logarithm of concentration:

$$A3)\ RF_{CO2} = 6.3 \ln \frac{c_t}{c_0}$$

where *c* is atmospheric concentration, *t* is the future year in question, and *0* refers to pre-industrial concentration (280 parts per million).[21]

Noting that several studies had found a smaller radiative forcing for $2xCO_2$, Myhre et al (1998) applied three models to arrive at a revised parameter of 5.35 (p. 2718). The Third Assessment Report (TAR) of the IPCC in 2001 adopted this parameter (IPCC, 2001, 356-67). Applying these two respective parameter values yields radiative forcing for $2xCO_2$ at 4.37 Wm^{-1} and 3.71 Wm^{-1} respectively.[22]

The four possible combinations of the alternative values for the parameter in equation A3) and the parameter corresponding to $4\, \sigma T^3$ (the inverse of the derivative of equation A.1) yield the following direct warming effect from $2XCO_2$ forcing: 1.16°, 0.99°, 1.35°, and 1.15°, with

maintain total albedo even with no atmosphere. The albedo of snow is a central value of 0.65, and of sea ice, 0.5 (Lewis, Weaver, and Aby, 2006, 1-2). Albedo of bare land could be 0.12 based on that of the moon (Saari and Shorthill, 1972, 173), and albedo of the ocean is 0.06 (NSIDC, 2020). With land 29 percent of earth's surface and oceans 71 percent (Pidwirny, 2006), coverage of 50 percent of land by snow and 50 percent of ocean area by ice would provide albedo of 0.31.

[21] IPCC (1990), p. 56.

[22] That is: multiplying by the natural logarithm of 2, or 0.693.

an average of 1.16°.[23] On this basis, downward revisions of radiative forcing from $2xCO_2$ appear to be approximately offset by upward revisions in the estimated warming per Wm^{-2} of radiative forcing. With respect to the Equilibrium Climate Sensitivity (ECS) for warming from a doubling of carbon dioxide above pre-industrial levels, to arrive at the long-standing central estimate of 3°C would correspondingly imply a positive feedback multiplier lying in a range of 2.22 to 3.03, with an average of 2.61.

[23] With FAR for IPCC (1990), TAR for IPCC (2001), SB for Stefan-Boltzmann and CM for climate models (Bony et al); and with the first subscript being degrees warming per Wm^{-2} and the second subscript being radiative forcing from $2xCO_2$, these combinations are respectively: SB,FAR; SB,TAR; CM,FAR; CM,TAR.

Appendix B

Scenario RCP8.5 in the Fifth Assessment Report

In its Fifth Assessment Report, the IPCC shifted from emission scenarios based on socioeconomic storylines to scenarios designed to yield specific alternative thresholds of radiative forcing by 2100, with its highest scenario reaching 8.5 Wm^{-2}. In this scenario, annual emissions of carbon dioxide rise from about 37 $GtCO_2$ (about 10 GtC) in 2010 to a high plateau of 106 $GtCO_2$ (29 GtC) by 2100. Emissions of methane rise from about 7 gigatons of carbon-dioxide equivalent ($GtCO_2eq$) to about 17 $GtC0_2eq$ in 2100. Emissions of nitrous oxide rise from about 4 $GtCO_2eq$ in 2010 to about 10 $GtCO_2eq$ in 2100. Aggregate $GtCO_2eq$ emissions of Kyoto-protocol greenhouse gases rise from about 40 $GtCO_2eq$ to about 132 $GtCO_2eq$ in 2100 (Meinshausen et al, 2011, p. 228).[24] The ratio of carbon dioxide to total greenhouse gas emissions in CO_2 equivalent remains approximately unchanged, rising slightly from 78 percent to 80 percent.

Scenario RCP8.5 in AR5 closely tracks scenario SRES-A2 used in the third and fourth assessment reports (IPCC, 2001, 2007). (SRES refers to Special Report on Emissions Scenarios.) AR5 reports that by 2100 both RCP8.5 and SRES-A2 reach 8 wm^{-2} radiative forcing, whereas two other SRES scenarios only reach 6 wm^{-2} (A1B) or 4 wm^{-2} (B1). (IPCC, 2013, 1046). These three SRES scenarios represent, respectively:

> … a very heterogeneous world [SRES-A2]. The underlying theme is self-reliance and preservation of local identities. Fertility patterns across regions converge very slowly, which results in continuously increasing global population. Economic development is primarily regionally oriented and per capita economic growth and technological change are more fragmented and slower than in other storylines. … [In related scenario A1B there is] … very rapid economic growth, global population that peaks in mid-century and declines thereafter, and the rapid introduction of new and more efficient technologies … [with a balance across both] fossil intensive …[and] non-fossil energy sources. [In scenario B1 there are] … rapid changes in economic structures toward a service and information economy, with reductions in material intensity, and the introduction of clean and resource-efficient technologies. (Nakićenović and Swart, 2000, 4-5).

The Fifth Assessment Report is the first to extend the horizon beyond 2100, and incorporates projections through 2300. The 2100-2300 “Extended Concentration Pathways” using:

[24] The other Kyoto greenhouse gases are Hydrofluorocarbons (HFCs), Perfluorocarbons (PFCs), and Sulphur hexafluoride (SF_6).

> … simple assumptions about GHG and aerosol emissions and concentrations beyond 2100 (such as stabilization or steady decline) … designed as hypothetical "what if" scenarios, not as an outcome of an IAM assuming socioeconomic considerations … RCP8.5 assumes stabilization with high emissions between 2100 and 2150, then a linear decrease until 2250 (IPCC, 2013, 1047).

The underlying study (Meinshausen et al, 2011, 226-27) reports that RCP8.5 features "constant emssions after 2100, followed by a smooth transition to stabilized concentrations after 2250 achieved by linear adjustment of emissions after 2150." This path leads to CO_2 stabilization after 2250 at approximately 2000 ppm (4,240 GtC atmospheric stock).

More specifically, carbon dioxide emissions plateau at 106 $GtCO_2$ annually during 2100-2150, and then decline linearly to a much lower plateau at 11 $GtCO_2$ starting in 2250. In contrast, annual methane emissions remain constant at their 2100 peak of about 17 $GtCO_2$-eq during the full period 2100-2300. This contrast reflects the much shorter atmospheric residence of methane than CO_2 combined with the shift from constant emissions in 2100-2150 to "stabilized concentrations" after 2250. As a consequence, the share of carbon dioxide in total annual CO_2-equivalent emissions falls from 80 percent in 2100-2150 to 31 percent by 2250 and after (Meinshausen et al, 2011, 288). However, as indicated in the main text, the decline of the broad TCRE parameter τ from about 1.7^oC/1000 GtC cumulative emissions in 2100 to about 1.5^oC/1000 GtC cumulative emissions by 2300 implies a rising share of CO_2 in the effective radiative forcing, which depends on atmospheric concentrations rather than then-current emissions. As discussed in Appendix D on the radiative forcing of methane and nitrous oxide, application of the IPCC (2013, 8SM-7) formulas for radiative forcing to concentration paths in RCP8.5 confirms a rising relative influence of carbon dioxide in the combined radiative forcing of these three most important greenhouse gases over this period.

Appendix C

Influences Turning the Marginal Warming Effect of the Emissions Path from Falling to Rising

The logarithmic relationship of radiative forcing and hence warming to the atmospheric concentration of carbon dioxide might seem to imply that, especially if annual emissions remain constant rather than rising significantly, future warming is likely to remain within relatively benign limits. That intuitive diagnosis is incorrect. This appendix decomposes three effects that explain this paradox: the *legacy* effect, the *atmospheric retention* effect, and the *rising emissions* effects.

The legacy effect addresses the fact that new annual emissions are relevant only insofar as they increase the atmospheric stock of carbon dioxide. That stock does not start out at zero, so even though the logarithm of a series comprising solely the summation of successive annual emissions would show a rapid dropoff in the marginal increase, the corresponding marginal logarithmic series for atmospheric stock shows very little dropoff. As discussed in the main text, an increase in the atmospheric retention rate can raise the marginal pace of the logarithm of atmospheric stock. Finally, an exponentially rising path of emissions violates the assumption of constant annual emissions that implicitly underlies the intuition of falling marginal warming.

Table C.1 illustrates these three effects. The first "Simple" panel reflects the numerical relationships implicit in the intuition of benignancy. The focus is solely on cumulation of prospective emissions at a constant rate. Emissions are set a constant rate of 10 GtC annually, the rate in 2010. In the simplest formulation, in which the retention rate is 100 percent, the cumulative stock correspondingly rises by 10 GtC annually. The natural logarithm of this stock rises from 2.303 at the end of the first year to 2.996 at the end of the second, an increment of 0.693 (final row in the panel). By the tenth year, the logarithm rises from 4.5 to 4.605, an increase of only 0.105. The change in the logarithm of the stock indeed falls off dramatically over time.

The second panel shows the legacy effect of the existing atmospheric stock, which stood at about 823 Gtc in 2010 (comprising 594 GtC pre-industrial and 229 anthropogenic). Emissions again are 10 GtC annually, and in this more realistic case have an atmospheric retention rate of 50 percent. In this scenario the natural logarithm of the stock rises, but very slowly. Although the marginal change in the logarithm still declines, it does so at a minimal pace, falling from 0.0056 in year 2 to 0.0054 by year 10. An implication is that *calibration against the existing legacy of atmospheric carbon turns the marginal warming effect of a constant stream of emissions nearly constant*, leading to near-linear warming.

Table C.1
Logarithm of Carbon Stock: Legacy, Retention, and Emissions-growth Influences

Year	1	2	3	4	5	6	7	8	9	10
Simple										
Flow	10	10	10	10	10	10	10	10	10	10
Stock	10	20	30	40	50	60	70	80	90	100
ln stock	2.303	2.996	3.401	3.689	3.912	4.094	4.248	4.382	4.500	4.605
△ ln stock		0.693	0.405	0.288	0.223	0.182	0.154	0.134	0.118	0.105
Legacy effect										
Flow	10	10	10	10	10	10	10	10	10	10
Stock	828	833	838	843	848	853	858	863	868	873
ln stock	6.719	6.725	6.731	6.737	6.743	6.749	6.755	6.760	6.766	6.772
△ ln stock		0.0060	0.0060	0.0059	0.0059	0.0059	0.0058	0.0058	0.0058	0.0057
Plus △ atmospheric retention										
Flow	10	10	10	10	10	10	10	10	10	10
Stock	828	833	838	843	848	855	862	869	876	883
ln stock	6.719	6.725	6.731	6.737	6.743	6.751	6.759	6.767	6.775	6.783
△ ln stock		0.0060	0.0060	0.0059	0.0059	0.0082	0.0082	0.0081	0.0080	0.0080
Plus △ emissions										
Flow	10.0	10.2	10.5	10.7	11.0	11.3	11.5	11.8	12.1	12.4
Stock	828	833	838	844	849	857	865	873	882	891
ln stock	6.719	6.725	6.731	6.738	6.744	6.754	6.763	6.773	6.782	6.792
△ ln stock		0.0062	0.0063	0.0064	0.0065	0.0092	0.0094	0.0095	0.0097	0.0098

Source: author's calculations

In the next panel the influence of rising atmospheric retention is added. In year 6, the retention rate rises from 50 percent to 70 percent.[25] In this case, the change in the logarithm of atmospheric stock rises from about 0.006 annually in the first 5 years to about 0.008 in the second five-year period (although it declines very slightly over time within each sub-period). So *rising atmospheric retention can turn the path of marginal warming from slowly declining to rising.*

The final panel adds the reality of rising emissions. Annual emissions rise from 10 GtC in year 1 (the 2010 rate) to 12.4 GtC by year 10 (the RCP8.5 level for 2020). As in the previous panel, the atmospheric retention rate is 50 percent in the first half of the period and 70 percent in the second half. In this scenario there is an annual increase in the rate of change of the logarithm of atmospheric stock, which rises from 0.0062 in year 2 to 0.0092 in year 6 (as retention rises) and 0.0098 by year 10.

Overall, whereas the pace in the change of the logarithm of stock declines sharply from 0.69 in year 2 to 0.10 in year 10 in the simple framework ignoring the legacy of existing atmospheric carbon and assuming emissions remain flat at 10 GtC, it reverses to a path of acceleration from about 0.006 in year 2 to about 0.01 by year 10 once the pre-existing stock, rising retention rate, and rising emissions path are taken into account. Incorporation of the legacy effect alone broadly eliminates the basis for the intuitive "benign outlook" that might be inferred based solely on diminishing marginal warming thanks to merely logarithmic radiative forcing.

[25] For illustrative purpose. As shown in the main text, this shift is not projected until about 2060.

Appendix D

Radiative Forcing of Methane and Nitrous Oxide

The main text finds that the broad TCRE coefficient τ declines from 1.67°C per 1000 GtC of cumulative carbon dioxide in 2100 to 1.50°C by 2300. A rise in the share of carbon dioxide in total greenhouse gas forcing appears to be at least partially responsible for this decline.

Supplementary material to Chapter 8 of IPCC (2013, 8SM-7) provides the following formula for radiative forcing of the two most important other greenhouse gases: methane and nitrous oxide.[26] Their concentrations have an interactive effect. For methane:

$$D.1)\ \Delta F_M = 0.036\ (M_t^{0.5} - M_0^{0.5}) - [f(M_t, N_{pd}) - f(M_0, N_{pd})]$$

where ΔF_M is the change in radiative forcing, *M* is the atmospheric concentration of methane, *N* is the atmospheric concentration of nitrous oxide, and subscripts are *t* for the future year in question, *0* for pre-industrial, and *pd* for "present-day" or 2010 at the time of the report. The expression *f*(…) refers to the following function:

$$D.2)\ f = 0.47\ \ln\,[1 + 2.01 \times 10^{-5}(MN)^{0.75} + 5.31\ \times 10^{-15} M(MN)^{1.52}]$$

For nitrous oxide, the corresponding change in radiative forcing is:

$$D.3)\ \Delta F_N = 0.12\ (N_t^{0.5} - N_0^{0.5}) - [f(M_{pd}, N) - f(M_{pd}, N_0)]$$

Table D.1 shows the estimates of radiative forcing above pre-industrial levels for these three gasses that result from application of the IPPC formulas (with 5.35 as the coefficient for CO_2; see Appendix A). The share of CO_2 in the total for the three gases rises from 81 percent in 2100 to 86 percent in 2300.

Table D.1
Concentrations and Radiative Forcing above Pre-Industrial Levels for CO_2, CH_4, and N_2O
(Scenario RCP8.5)

Concentrations	Pre-industrial	2010	2100	2200	2300
CO_2 (ppm)	278	388.1	959	1825	1960
CH_4 (ppb)	722	1800	3800	3480	3460
N_2O (ppb)	270	324	435	511	526
△Radiative forcing (W/msq)					
CO_2		1.78	6.62	10.07	10.45
CH_4		0.48	1.07	0.99	0.99
N_2O		0.13	0.47	0.68	0.72

Source: IPCC (2013); Meinshausen et al (2011, 232); and author's calculations

[26] Also see Myhre et al, 1998, 2718.

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